**Section 4.4—Exponential and Logarithmic Equations**

**Exponential Equation**—an equation containing a variable in an exponent

All exponential functions are one-to-one; that is no two different ordered pairs have the same second component.

**Solving Exponential Equations by Expressing Each Side as a Power of the Same Base**

If , then M = N.

1. Rewrite the equation in the form
2. Set M = N.
3. Solve for the variable.

**Example**—Solve

Most exponential equations cannot be rewritten so that each side has the same base.

**Using Natural Logarithms to Solve Exponential Equations**

1. Isolate the exponential expression
2. Take the natural logarithm on both sides
3. Simplify using one of the following

or

1. Solve for the variable

**Example**—Solve the following

**Logarithmic Equation**—an equation containing a variable in a logarithmic expression

**Using the Definition of a Logarithm to Solve Logarithmic Equations**

1. Express the equation in the form
2. Use the definition of a logarithm to rewrite the equation in exponential form:

means

1. Solve for the variable
2. Check proposed solutions in the original equation. Include in the solution set only values for which M > 0.

**Example**—Solve:

Logarithmic expressions are defined only for logarithms of positive real numbers.

Always check proposed solutions of logarithmic equation in the original equation. Exclude values that give you the log of a negative number or a log of 0.

To rewrite as , we have to have a single logarithm whose coefficient is one.

**Example**—Solve .

**Using the One-to-One Property of Logarithms to Solve Logarithmic Equations**

1. Express the equation in the form . (Remember: you must have a single logarithm whose coefficient is 1 on each side of the equation.)
2. Use the one-to-one property to rewrite the equation without logarithms: If then M = N.
3. Solve for the variable.
4. Check in the original equation. Include only the values for which M > 0 and N > 0.

**Example**—Solve